



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

Solution by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

Let any given orthogonal system of circles be inverted with respect to a circle whose center is an intersection of two circles, one from each family, and neither of them a real circle. The result is a new orthogonal system containing two straight lines derived from the two circles, and each line is the locus of centers of the 'opposite' family of circles. Using these lines as axes of coördinates, the two circle families are  $(x-a)^2 + y^2 = c^2$ ,  $x^2 + (y-b)^2 = d^2$ , in which, because the circles are orthogonal,  $a^2 + b^2 = c^2 + d^2$ . Writing  $c^2 = a^2 - k^2$  in the last equation gives  $d^2 = b^2 + k^2$ , and the circle families become  $x^2 + y^2 - 2ax + k^2 = 0$ ,  $x^2 + y^2 - 2by - k^2 = 0$ .

The constants are now independent, but since any circle of one family is orthogonal to all of the other family it follows that  $a$  and  $b$  are the respective parameters. If now  $a$  and  $b$  are replaced by  $k \coth 2v$  and  $-k \cot 2u$ , respectively, the system may be written  $u + vi = \tan^{-1} \frac{x + yi}{k}$ , which shows that it is isothermal. Thus the given system is also isothermal, since it may be obtained from this one by inversion. When  $k$  is 0 or  $\infty$  the corresponding result is

$$u + vi = \frac{1}{x + yi}, \text{ or } u + vi = x + yi.$$

274. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

If a straight line  $AB$  is placed between two intersecting straight lines  $MN$  and  $PQ$  and is made to revolve through all possible positions having  $A$  always in  $MN$  and  $B$  always in  $PQ$ , what is the locus of any point  $L$  in  $AB$  or  $AB$  produced?

I. Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

We can choose coördinate axes so that the equations to the given lines are  $y = rx$ ,  $z = a$ ;  $y = -rx$ ,  $z = -a$ . Let the coördinates of  $A$ ,  $B$ ,  $L$  be, respectively,  $(h, rh, a)$ ,  $(k, -rk, -a)$ ,  $(x, y, z)$ . Then

$$\frac{x-h}{x-k} = \frac{y-rh}{y+rk} = \frac{z-a}{z+a} = \frac{AL}{BL} = m, \text{ say.}$$

$$\therefore h - mk = x(1-m) \dots \dots \dots (1), \quad r(h + mk) = y(1-m) \dots \dots \dots (2), \quad z(1-m) = a(1+m) \dots \dots \dots (3).$$

Hence the locus lies in a plane parallel to  $z=0$ , or to the given lines as is otherwise evident. Also  $AB^2 = l^2 = (h-k)^2 + r^2(h+k)^2 + 4a^2 \dots \dots \dots (4)$ . Eliminating  $h$ ,  $k$  between (1), (2), (4) we have an ellipse for the required locus, its equation being

$$(1-m)^2 \{ [y(m-1) + rx(m+1)]^2 + [y(m+1) + rx(m-1)]^2 r^2 \} = 4m^2 r^2 (l^2 - 4a^2).$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let  $O$  be the intersection of  $MN$ ,  $PQ$ .  $OA=a$ ,  $OB=b$ . Draw  $LD$  parallel to  $OB$ , and let  $D$  be in  $MN$ . Let  $OD=u$ ,  $DL=v$ ,  $AB=c$ ,  $AL=d$ ,  $\angle AOB=\beta$ . Then  $a^2 + b^2 - 2ab\cos\beta = c^2$ ;  $c : d = a : a \pm u$ ;  $c : d = b : v$ .

$$\text{Hence } a = \pm \frac{cu}{d-c}, \quad b = \frac{cv}{d}, \quad \text{and } \frac{c^2 u^2}{(d-c)^2} + \frac{c^2 v^2}{d^2} \mp \frac{2c^2 uv \cos \beta}{d(d-c)} = c^2.$$

$$\therefore \frac{u^2}{(d-c)^2} + \frac{v^2}{d^2} \mp \frac{2uv \cos \beta}{d(d-c)} = 1.$$

$\therefore$  The locus is an ellipse.

III. Solution by A. H. HOLMES, Brunswick, Maine.

Suppose the straight lines  $MN$  and  $PQ$  intersect each other at right angles at  $O$ , and  $AB$  placed between them:  $A$  on  $MN$  and  $B$  on  $PQ$ , and  $L$  a point in  $AB$ . Draw  $LO$ . Put  $AL=b$ ,  $BL=a$ , and  $LO=r$ , and  $LAO=\phi$ ,  $AOL=\theta$ . Then  $b\sin\phi=r\sin\theta$ , and  $a\cos\phi=r\cos\theta$ .

$$\therefore \sin^2\phi = \frac{r^2}{b^2}\sin^2\theta, \text{ and } \cos^2\phi = \frac{r^2}{a^2}\cos^2\theta. \quad \therefore r = \frac{ab}{\sqrt{(a^2\sin^2\theta + b^2\cos^2\theta)}}.$$

Therefore the locus of point  $L$  is an ellipse whose semi-major axis is  $BL$  and whose semi-minor axis is  $AL$ . When  $MN$  and  $PQ$  intersect obliquely at angle  $\psi$  the semi-minor axis would be  $\frac{ab\sin\psi}{\sqrt{(a^2 - b^2\cos^2\psi)}}$ .

Also solved by R. D. Carmichael, and J. Scheffer.

275. Proposed by PROFESSOR WILLIAM HOOVER, Ph. D., Athens, Ohio.

An hyperbola is drawn touching the axes of an ellipse, and the asymptotes of the hyperbola touch the ellipse. Prove that the center of the hyperbola lies on one of the equal conjugate diameters of the ellipse.

Solution by the PROPOSER.

Let  $(x', y')$  be the intersection of the tangents to the ellipse  $a^2y^2 + b^2x^2 - a^2b^2 = 0$  ..... (1); then these tangents being the asymptotes of the hyperbola,  $(x', y')$  is the center of the hyperbola. The equation to the tangents to (1) from  $(x', y')$  is

$$(a^2y^2 + b^2x^2 - a^2b^2)(a^2y'^2 + b^2x'^2 - a^2b^2) = (a^2y'y + b^2x'x - a^2b^2)^2 \text{ ..... (2),}$$

$$\text{or, } (y'^2 - b^2)x^2 + (x'^2 - a^2)y^2 - 2x'y'xy + 2b^2x'x + 2a^2y'y - (a^2y'^2 + b^2x'^2) = 0 \text{ ..... (3).}$$

Now, the equation to the asymptotes of a conic differs from the equation to the conic by a constant only; then adding  $c$  to the left member of (3) we have the equation to the hyperbola.

If now  $y=0$  in this equation to the hyperbola, we have